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ON THE DYNAMICS OF THE TRANSMISSION WITH A DOUBLE CARDAN JOINT

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1 Introduction

Although the cardan joint receives many industrial applications, its dynamics is not yet accurately described by the mathematical models found in technical literature. One of the most sophisticated modeling of the dynamics of such a joint is presented in a series of papers by F. Freudenstein and his coworkers [Fischer 1984, Chen 1986, Freudenstein 1990].

In the named approach the analysis equations follow from the fundamental laws of rigid body mechanics. Thus, it cannot be applied to the real topology of the cardan joint, but only to an *equivalent* RCCC linkage. It is well known that the cardan joint is an overconstrained linkage and that, in such cases, joint reactions can be reliably computed only by taking into account the elasticity effects. Purpose of our research is the improvement of current dynamic models in the field of transmission joints. In a previous paper [Biancolini et al. 1998] an integrated multibody/FEM approach is used for the dynamic analysis of a cardan transmission.

In this paper is studied the dynamics of the widely used transmission formed of two series connected cardan joints.

The paper will analyze the influence of the intermediate shaft elasticity on the overall performances of the transmission.

This paper reports a closed form solution of the torque analysis problem in a double cardan joint. In the past a similar analysis was described only by F. Duditza[Duditza 1971], but was limited to static conditions. Our treatment includes the effects of inertia.
Moreover, using a model with lumped elasticity, mass and inertia, the analysis of the system was performed with a flexible multibody approach. The results emphasize that both torque and bearing reactions are altered by the shaft stiffness and are related to the the angular velocity.

2 Review of previous contributions

At the book level, the most relevant sources of informations on the cardan joint are the book authored by F. Duditza [Duditza 1971] and the handbook edited by E.R. Wagner [Wagner 1979]. The book of Duditza, originally published in 1966 and translated in many languages, contains the description of different mathematical models for kinematic, dynamic, vibrational and stress analysis of polycardan mechanisms. Although the kinematics seems extensively studied, the literature on the models for internal forces analysis seems sparse. The first studies on the dynamics of the cardan joint appear around 1930-1940 and may have been prompted by the numerous joint failures observed in the drivelines of automobiles. These studies hinted the presence of *rocking torques* or *secondary couples*, with direction orthogonal both to input and output shafts, as causes of the failures. Refined models of static analysis, based on the use of the algebra of dual numbers and dual matrices [Yang 1965, Fischer 1984], confirmed the findings.

Investigations concerned with joint vibration are due to [Rosenberg 1958] and, more recently, to [Sheu et al. 1996].
In a previous paper ([Biancolini et al. 1998]) the authors proposed a model for the flexible multibody analysis of a single cardan joint.

3 Nomenclature

- I_{ix}, I_{iy}, I_{iz}: principal moment of inertia of the i^{th} body;
- M_{ik}: torque acting from the i^{th} to the k^{th} body;
- M_a, M_u: applied and resisting torques;
- ϑ_1, ϑ_3, ϑ_5 angular positions of input, intermediate and output shafts, respectively (see figure 6);
- $x_i y_i z_i$ moving cartesian system attached to the i^{th} body (see figure 1)
- $X_k Y_k Z_k$ fixed cartesian system associated to the k^{th} body (see figure 1)
- ω_i: angular velocity of the i^{th} body, measured in the cartesian system $o - x_i y_i z_i$.

Dots denote differentiation w.r.t. time.

4 Torque analysis-Rigid bodies

The proposed analysis of torques in a double cardan joint is based on the following hypotheses

- rigid bodies;
- absence of dissipative effects, manufacturing and/or mounting defects;
- input, intermediate and output shafts are connected with the frame with rotoidal joints;
- constant angular velocity of the input shaft.

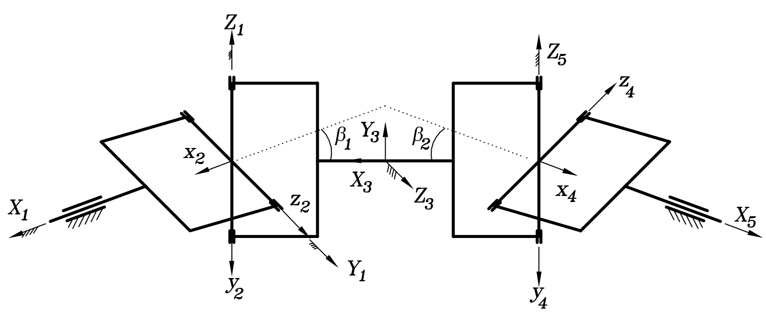

Figure 1: Scheme of a double-cardan joint.

The kinematics of a cardan joint is described in several sources (e.g. [Freudenstein 1990]) and will be not repeated here. To find the torques acting on each link, we use Euler's moment equations for motion with a fixed center of mass, the axes being principal axes:

- Input shaft (Link 1)

$$M_a + M_{21x} = I_{1x}\dot{\omega}_{1x} \tag{1}$$

- First cross (Link 2)

$$\begin{aligned}
M_{12x} + M_{32x} &= I_{2x}\dot{\omega}_{2x} + \omega_{2y}\omega_{2z}\left(I_{2z} - I_{2y}\right) && (2)\\
M_{12y} + M_{32y} &= I_{2y}\dot{\omega}_{2y} + \omega_{2z}\omega_{2x}\left(I_{2x} - I_{2z}\right) && (3)\\
M_{12z} + M_{32z} &= I_{2z}\dot{\omega}_{2z} + \omega_{2x}\omega_{2y}\left(I_{2y} - I_{2z}\right) && (4)
\end{aligned}$$

- Intermediate shaft (Link 3)

$$M_{23x} + M_{43x} = I_{3x}\dot{\omega}_{3x} \tag{5}$$

- Second cross (Link 4)

$$\begin{aligned} M_{54x} + M_{34x} &= I_{4x}\dot{\omega}_{4x} + \omega_{4y}\omega_{4z}\left(I_{4z} - I_{4y}\right) && (6)\\ M_{54y} + M_{34y} &= I_{4y}\dot{\omega}_{4y} + \omega_{4z}\omega_{4x}\left(I_{4x} - I_{4z}\right) && (7)\\ M_{54z} + M_{34z} &= I_{4z}\dot{\omega}_{4z} + \omega_{4x}\omega_{4y}\left(I_{4y} - I_{4z}\right) && (8) \end{aligned}$$

- Output shaft (Link 5)

$$M_{45x} + M_u = I_{5x}\dot{\omega}_{5x} \tag{9}$$

Under our hypotheses, rotoidal joints cannot transmit torques. Thus the following equalities must hold: M_{32y}=M_{12z}=M_{34y}=M_{54z}=0. Moreover, the following transforms must be taken into account (see Appendix) $\{M_{45}\} = -[T_{45}]\{M_{54}\}$, $\{M_{21}\} = -[T_{21}]\{M_{12}\}$, $\{M_{23}\} = -[T_{23}]\{M_{32}\}$, $\{M_{43}\} = -[T_{43}]\{M_{34}\}$. Once the kinematics and the resisting torque M_u are specified, we obtain a linear system of 25 scalar equations where the unknowns are the components of all the eight shaking moments considered plus the applied torque M_a. Since the matrix is sparse, the system of equations can be easily solved by hand.

The numerical data for this example, in SI units, are as follows: M_u=750, I_{1x}=I_{5x}= 0.01528, I_{2y}=I_{4y}=I_{2z}=I_{4z}=0.00111, I_{2x}=I_{4x}=0.00202. The plots of the applied torque M_a are shown in figure 2 for different angular speeds of the driving shaft. This values have been extrapolated from the dimensions reported in a catalog of commercial double cardan joints. The value of the adopted value of resisting torque is recommended by the catalog for the following working conditions: β_1=β_2=10 deg, ω_2= 2000 r.p.m. (shaft life:5,000 hours).

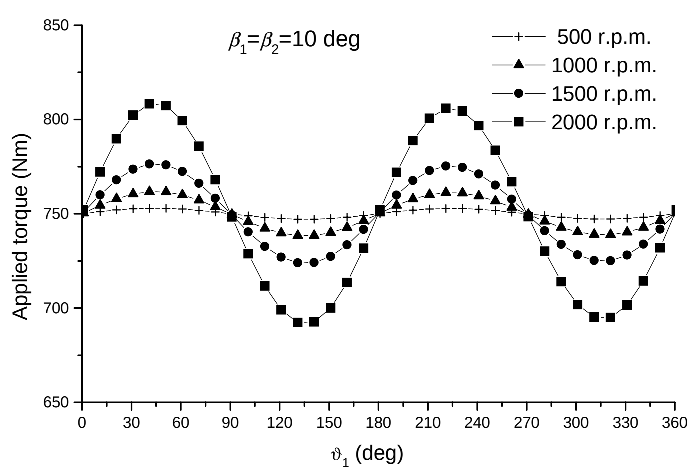

Figure 2: Applied torque.vs. angular position and different angular speeds of the driving shaft.

5 Flexible model

In a previous paper [Biancolini et al. 1998], it was shown that if the single cardan joint operates far from resonance, in the dynamic analysis the contribution of the links deformability can be neglected. Under this hypothesis the behaviour of the double cardan joint was studied by means of a simplified model where:

- all the inertia properties of the elements are taken into account;

Table 1: Modal analysis of the double cardan joint (initial position)

Mode No.	Frequency	Vibration mode
1	127.1 Hz	bending of intermediate shaft
2	129.7 Hz	bending of intermediate shaft
3	347.5 Hz	bending of intermediate shaft
4	355.2 Hz	torsion of intermediate shaft
5	368.6 Hz	bending of intermediate shaft

- the elasticity was introduced only for the intermediate shaft as bending, axial and torsional deformation modes;
- absence of friction and manufacturing and/or mounting defects.

The software Working Model v. 4.0 was used as analysis tool. For the purpose of comparison, the kinematic structure and the dimensions of the linkage analysed are the same of those already reported at section 5. Forks and crosses are considered as rigid bodies and connected with cylindrical joints. With reference to figure 3, the intermediate shaft is discretized with 12 rigid bodies connected in series by means of elastic elements which allow both bending and torsion. The accuracy of this last submodel was checked by comparing its dynamic response with that obtained from a finite element model developed within the MSC/Nastran code. The vibration modes of the entire system in the initial position were computed with

Figure 3: Rendered view of the double cardan joint.

a finite element model. The results obtained for the first five modes have been summarized in Table 1.

The numerical results are recorded under steady state condition

The resisting torque is governed by the following algebraic equation $M_u = c\omega_5^2$, where $c = 0.017$ Nms^2/rad^2. Since the angular velocity of the output shaft is $\omega_5 \approx 2000$ r.p.m., the final value of the resisting torque is $M_u \approx 750$ Nm. During simulation the angular speed ω_2 of the input link is kept constant. The required input torque was recorded for the following cases: a) $\omega_2 = 2000$ r.p.m., b) $\omega_2 =$ 4000 r.p.m. The figures 4 and 5 show the required input torque as computed by means of the two models (*i.e.* rigid and flexible).

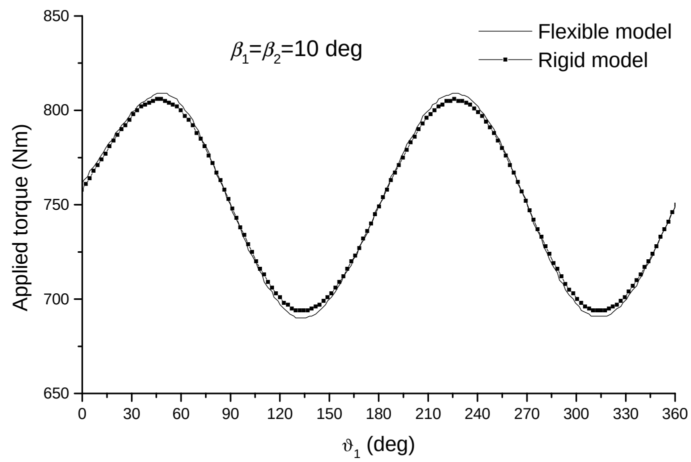

Figure 4: Comparison between flexible and rigid model: applied torque at 2000 r.p.m.

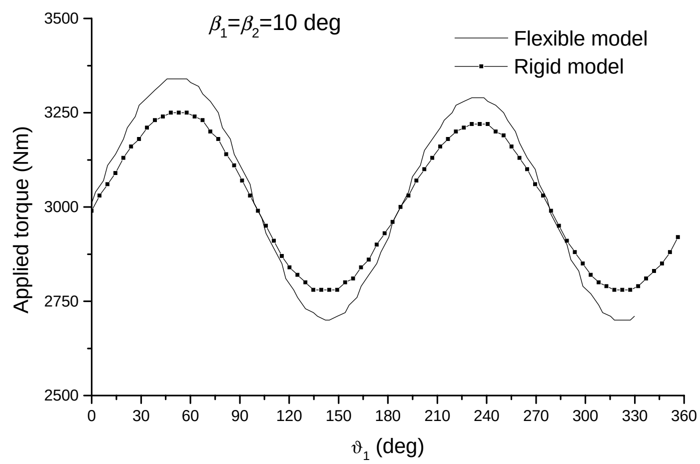

Figure 5: Comparison between flexible and rigid model: applied torque at 4000 r.p.m.

6 Conclusions

The paper discussed the influence of inertia and elasticity on the magnitude of shaking torques in a double cardan joint. The results of the closed form solution of torque analysis match those obtained using a commercial software for multibody analysis.
The numerical results summarized in figure 4 demonstrate that, within normal working conditions, there is a little influence of the elasticity on the input torque. However, at 4000 r.p.m., there is a significant difference between the results given by the rigid and by the flexible model. This speed is not recommended by the manufacturer of the joint analyzed.

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Appendix

The following matrices define the transformations between the different cartesian reference system ($c_i = \cos\vartheta_i$, $s_i = \sin\vartheta_i$, $c_{\beta k} = \cos\beta_k$, $s_{\beta k} = \sin\beta_k$).

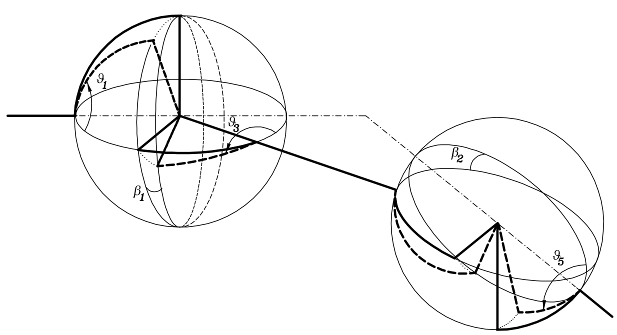

Figure 6: Convention for measuring angular positions of the links.

$$[T_{21}] = \begin{bmatrix} -s_1c_3c_{\beta1} - s_3c_1 & -c_3s_{\beta1} & 0 \\ s_1c_3s_{\beta1} & -c_3c_{\beta1} & c_1 \\ -c_1c_3s_{\beta1} & s_3 & s_1 \end{bmatrix} \quad , \quad [T_{13}] = \begin{bmatrix} c_{\beta1} & -s_{\beta1} & 0 \\ s_{\beta1} & c_{\beta1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ,$$

$$[T_{45}] = \begin{bmatrix} -s_5c_3c_{\beta2} - s_3c_5 & -c_3s_{\beta2} & 0 \\ s_5c_3s_{\beta2} & -c_3c_{\beta2} & c_5 \\ -c_3c_5s_{\beta2} & s_3 & s_5 \end{bmatrix} \quad , \quad [T_{53}] = \begin{bmatrix} -c_{\beta2} & s_{\beta2} & 0 \\ -s_{\beta2} & -c_{\beta2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ,$$

$$[T_{23}] = [T_{21}]\,[T_{13}] \quad , \quad [T_{34}] = [T_{53}]^T\,[T_{45}]^T \quad .$$